Primal

Dual

$$lpha = \sup_{X \geq 0} \left\{ \operatorname{Tr}[AX] : \Phi(X) \leq B
ight\}$$

Constrained forms

Weak duality: $\alpha \leq \beta$

$$eta = \inf_{Y \geq 0} \left\{ \operatorname{Tr}[BY] : \Phi^\dagger(Y) \geq A
ight\}$$

$$lpha = \sup_{X,W \geq 0} \left\{ \operatorname{Tr}[AX] : B - \Phi(X) = W
ight\}$$

Introduce slack variables

$$eta = \inf_{Y,Z \geq 0} \left\{ \operatorname{Tr}[BY] : \Phi^\dagger(Y) - A = Z
ight\}$$

$$lpha(c) = \sup_{X,W \geq 0} \left\{ ext{Tr}[AX] - c \|B - \Phi(X) - W\|_2^2
ight\}$$
 $lpha = \lim_{c o \infty} lpha(c)$

Unconstrained forms (introduce penalty terms)

$$eta(c) = \inf_{Y,Z \geq 0} \left\{ ext{Tr}[BY] + c ig\| \Phi^\dagger(Y) - A - Z ig\|_2^2
ight\}$$
 $eta = \lim_{c o \infty} eta(c)$

$$egin{aligned} lpha(c) &= \sup_{\substack{\lambda, \mu \geq 0, \
ho, \sigma \in \mathcal{D}}} \left\{ \lambda \operatorname{Tr}[A
ho] - c \|B - \lambda \Phi(
ho) - \mu \sigma\|_2^2
ight\} \ &lpha &= \lim_{c o \infty} lpha(c) \end{aligned}$$

Write positive semidefinite matrices as scaled density matrices

$$eta(c) = \inf_{egin{subarray}{c} \kappa,
u \geq 0, \ au, \omega \in \mathcal{D} \end{array}} \left\{ \kappa \operatorname{Tr}[B au] + c igg \| \kappa \Phi^\dagger(au) - A -
u \omega igg \|_2^2
ight\}$$
 $eta = \lim_{c o \infty} eta(c)$

$$ilde{lpha}(c) = \sup_{\substack{\lambda, \mu \geq 0, \ heta_1, heta_2 \in \Theta}} \left\{ \lambda \operatorname{Tr}[A
ho(heta_1)] - c \|B - \lambda \Phi(
ho(heta_1)) - \mu \sigma(heta_2)\|_2^2
ight\}$$

Parameterize density matrices

$$ilde{eta}(c) = \inf_{\substack{\kappa,
u\geq 0,\ heta_3, heta_4\in\Theta}} \left\{\kappa\operatorname{Tr}[B au(heta_3)] + cig\|\kappa\Phi^\dagger(au(heta_3)) - A -
u\omega(heta_4)ig\|_2^2
ight\}$$

 $eta \leq \lim_{c o \infty} ilde{eta}(c)$

 $\lim_{c o\infty} ilde{lpha}(c)\leq lpha$

Estimate the objective function using a quantum computer + Perform the optimization using a classical optimizer