
QSlack
Primal
Dual
$\alpha = \sup_{X \geq 0} \{ \mathrm{Tr}[AX] : \Phi(X) \leq B \}$
Constrained forms
Weak duality: $\alpha \leq \beta$
$\beta = \inf_{Y \geq 0} \{ \mathrm{Tr}[BY] : \Phi^{\dagger}(Y) \geq A \}$
$\alpha = \sup_{X,W \geq 0} \{ \mathrm{Tr}[AX] : B - \Phi(X) = W \}$
Introduce slack variables
$\beta = \inf_{Y,Z \geq 0} \{ \mathrm{Tr}[BY] : \Phi^{\dagger}(Y) - A = Z \}$
$\alpha(c) = \sup_{X,W \geq 0} \left\{ \mathrm{Tr}[AX] - c\|B - \Phi(X) - W\|_2^2 \right\}$
$\alpha = \lim_{c \to \infty} \alpha(c)$
Unconstrained forms (introduce penalty terms)
$\beta(c) = \inf_{Y,Z \geq 0} \left\{ \mathrm{Tr}[BY] + c\left\|\Phi^{\dagger}(Y) - A - Z\right\|_2^2 \right\}$
$\beta = \lim_{c \to \infty} \beta(c)$
$\alpha(c) = \sup_{\substack{\lambda,\mu \geq 0, \\ \rho,\sigma \in \mathcal{D}}} \left\{ \lambda \mathrm{Tr}[A\rho] - c\|B - \lambda\Phi(\rho) - \mu\sigma\|_2^2 \right\}$
$\alpha = \lim_{c \to \infty} \alpha(c)$
Write positive semi-definite matrices as scaled density matrices
$\beta(c) = \inf_{\substack{\kappa,\nu \geq 0, \\ \tau,\omega \in \mathcal{D}}} \left\{ \kappa \mathrm{Tr}[B\tau] + c\left\|\kappa\Phi^{\dagger}(\tau) - A - \nu\omega\right\|_2^2 \right\}$
$\beta = \lim_{c \to \infty} \beta(c)$
$\tilde{\alpha}(c) = \sup_{\substack{\lambda,\mu \geq 0, \\ \theta_1,\theta_2 \in \Theta}} \left\{ \lambda \mathrm{Tr}[A\rho(\theta_1)] - c\|B - \lambda\Phi(\rho(\theta_1)) - \mu\sigma(\theta_2)\|_2^2 \right\}$
$\lim_{c \to \infty} \tilde{\alpha}(c) \leq \alpha$
Parameterize density matrices
$\tilde{\beta}(c) = \inf_{\substack{\kappa,\nu \geq 0, \\ \theta_3,\theta_4 \in \Theta}} \left\{ \kappa \mathrm{Tr}[B\tau(\theta_3)] + c\left\|\kappa\Phi^{\dagger}(\tau(\theta_3)) - A - \nu\omega(\theta_4)\right\|_2^2 \right\}$
$\beta \leq \lim_{c \to \infty} \tilde{\beta}(c)$
Estimate the objective function using a quantum computer + Perform the optimization using a classical optimizer