QSlack: A Slack-Variable Approach for Variational Quantum Semi-Definite Programming

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Motivation

- Solving optimization problems is a key task for which quantum computers could possibly provide a speedup over the best known classical algorithms.
- Particular classes of optimization problems including semi-definite programming (SDP) have wide applicability in many domains of computer science, engineering, mathematics, and physics.
- Here we focus on semi-definite programs for which the dimensions of the variables involved are exponentially large, so that standard classical SDP solvers are not helpful for such large-scale problems.

Semi-Definite Programs & Duality

Primal SDP:

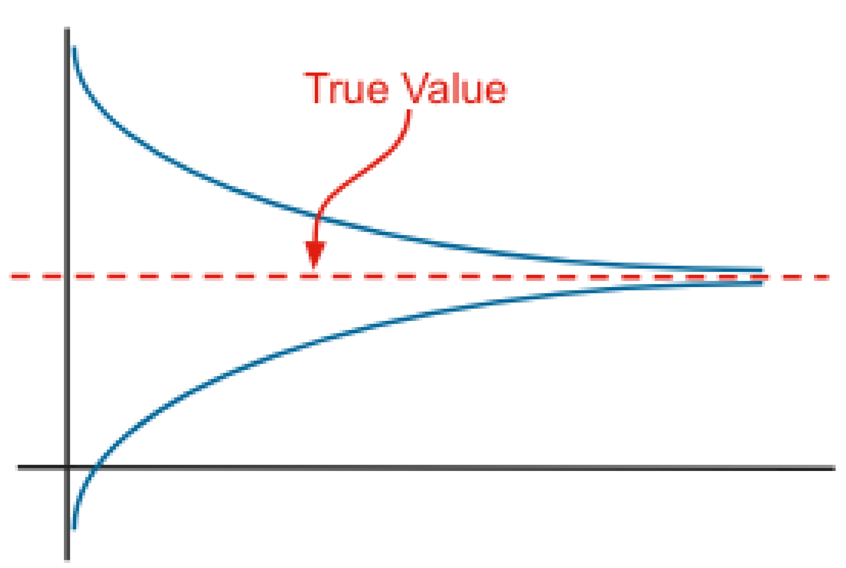
$$\alpha \coloneqq \sup_{X\geq 0} \left\{ \operatorname{Tr}[AX] : \Phi(X) \leq B \right\}.$$

Dual SDP:

$$\beta \coloneqq \inf_{Y\geq 0} \left\{ \operatorname{Tr}[BY] : \Phi^\dagger(Y) \geq A \right\}.$$

Strong Duality: $\alpha = \beta$.

Key Theoretical Contribution: By making use of SDP duality theory, the QSlack method provides a theoretical guarantee that the global optima of the objective functions sandwich the true optimal value from above and below.



QSlack Method Overview

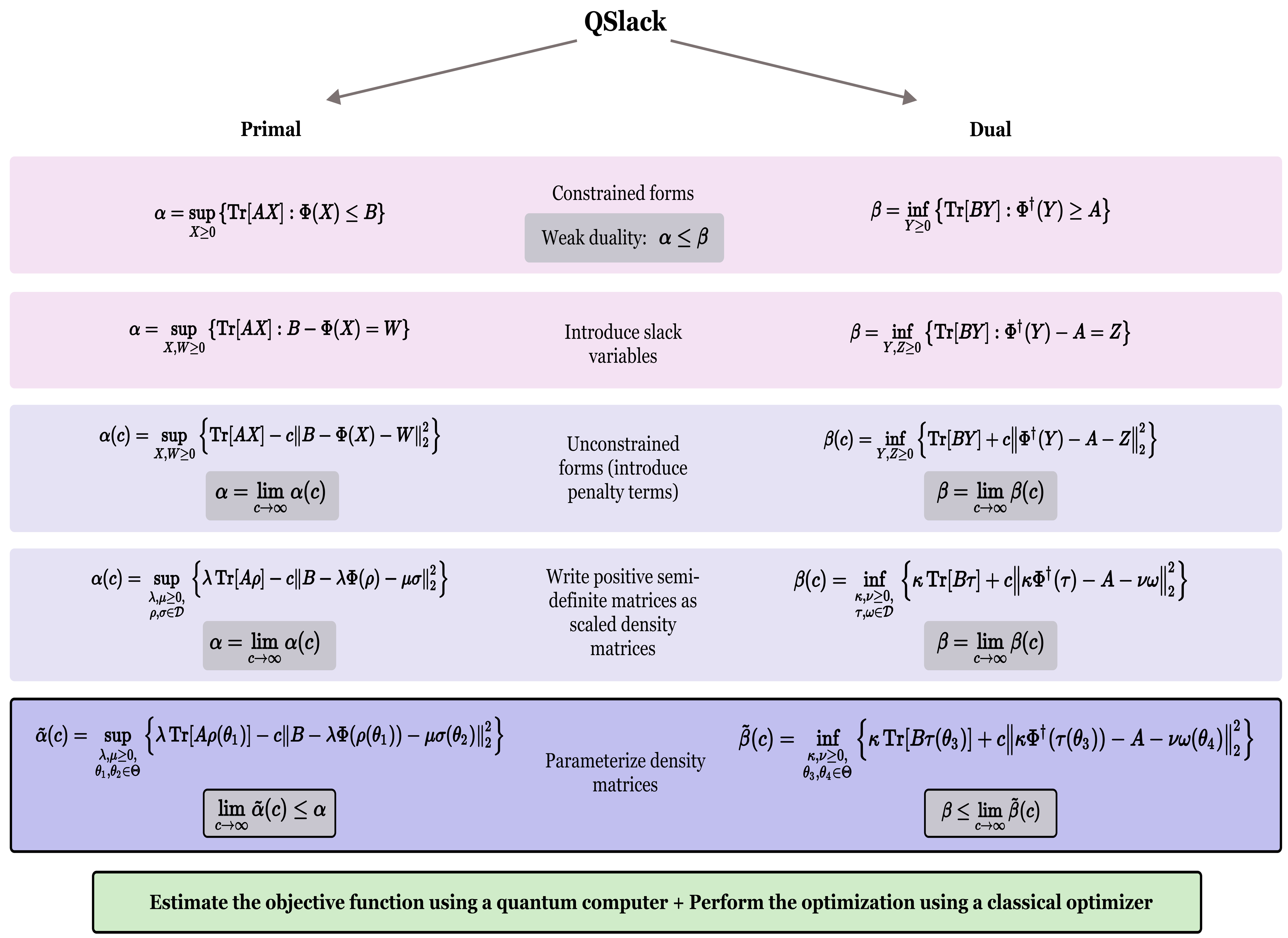


Figure 1. Overview of the QSlack method. From top to bottom, we modify the expressions of the primal and dual semi-definite programs to a form that can be evaluated on a quantum computer.

References

QSlack arXiv:2312.03830

Dual VQE arXiv:2312.03083

Example: Trace Distance

- Normalized trace distance of n-qubit states ρ and σ:

$$\frac{1}{2}\|\rho - \sigma\|_1 = \sup_{\Lambda\geq 0}\{\operatorname{Tr}[\Lambda(\rho-\sigma)] : \Lambda \leq I\} = \inf_{Y\geq 0}\{\operatorname{Tr}[Y] : Y \geq \rho - \sigma\}$$

- Prior approaches focus on the primal optimization to provide lower bounds on $\frac{1}{2}\|\rho - \sigma\|_1$.
- For primal optimization, one can use parameterized measurement circuits.
- Dual optimization can be reformulated using QSlack:

$$\inf_{Y\geq 0}\{\operatorname{Tr}[Y] : Y \geq \rho - \sigma\} = \lim_{c\to\infty} \inf_{\lambda,\mu\geq 0,\omega,\tau\in\mathcal{D}} \left\{\lambda + c\left\|\lambda\omega - \rho + \sigma - \mu\tau\right\|_2^2\right\}$$

- The optimization is performed over parameterized states, with estimation using destructive swap test or mixed-state Loschmidt echo test.

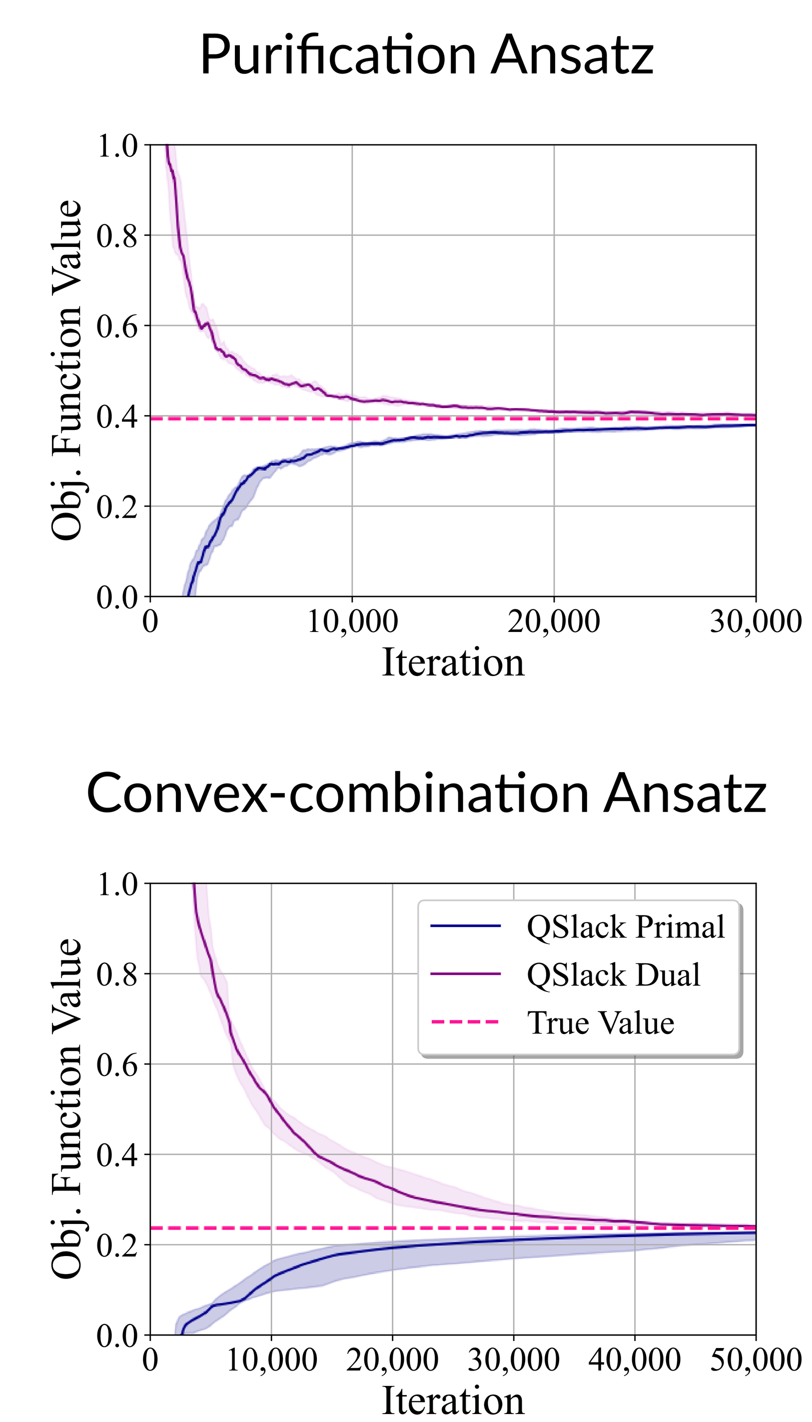


Figure 2. Convergence of the primal and dual optimizations to their optimal values in the Qslack Trace Distance example.