Testing Symmetry on Quantum Computers

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Joint work with M. Laborde and M. Wilde - arXiv:2105.12758

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- Symmetry defintion
  - Physical intuition
  - Mathematical notion
- Quantum algorithms to test symmetry
- Examples

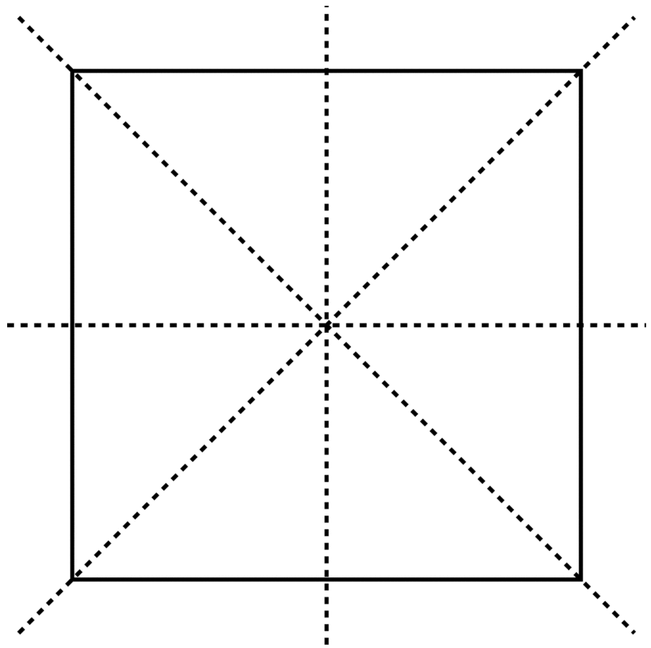
What are symmetries?

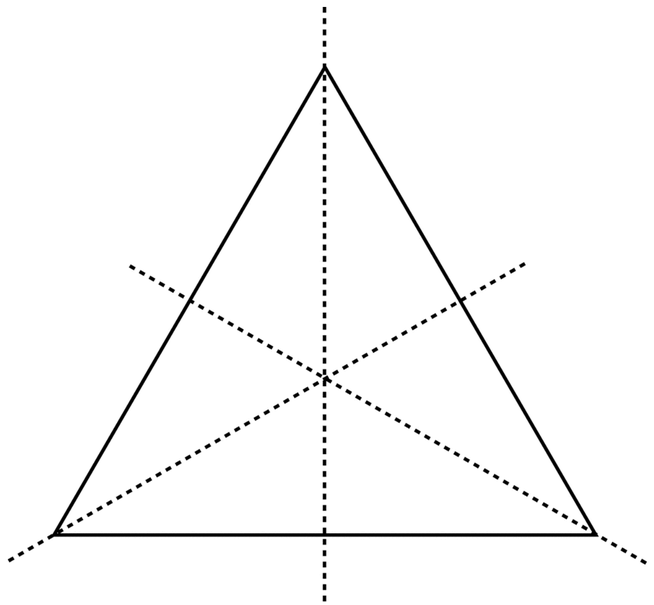
How do we test them?

What are symmetries?

Physical Intuition and Motivation

- Fundamental concept in physics and quantum information.
- ‘Unchanged’ upon transformation.





What are symmetries?

Physical Intuition and Motivation

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- ‘Unchanged’ upon transformation.
- Asymmetrical states are ‘useful’.

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Symmetry	Resource
Time evolution Hamiltonian	Time-Keeping
Extendibility	Entanglement
Rotations	Reference Frames

What are symmetries?

Mathematical Definition

Symmetry Operations

G-Bose Symmetry

$$\rho_S = U_S(g)\rho_S \;\; \forall g \in G$$

G-Symmetry

$$\rho_S = U_S(g)\rho_S U_S^\dagger(g) \;\; \forall g \in G$$

G-Bose Symmetric Extendibility

$$\omega_{RS} = U_{RS}(g)\omega_{RS} \;\; \forall g \in G$$
$$\rho_S = \mathrm{Tr}_R[\omega_{RS}]$$

G-Symmetric Extendibility

$$\omega_{RS} = U_{RS}(g)\omega_{RS}U_{RS}^\dagger(g) \;\; \forall g \in G$$
$$\rho_S = \mathrm{Tr}_R[\omega_{RS}]$$

How do we test them?

Measure of symmetry with respect to group

- Need a measure of symmetry - not just ‘Yes/No’.
- ‘Distance’ from closest symmetric state.

$$\max_{\sigma \in S} F(\rho, \sigma)$$

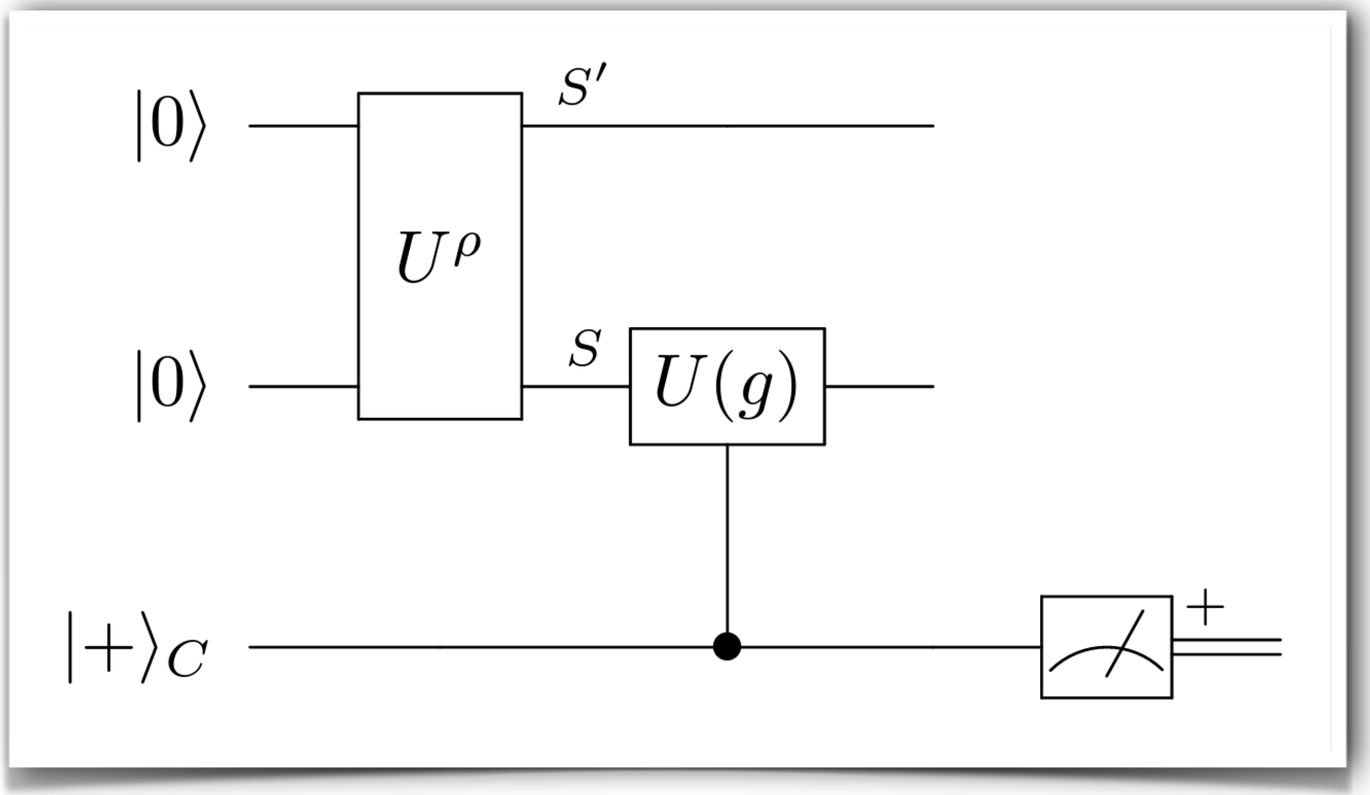
How do we test them?

G-Bose Symmetry

$$\rho_S = U_S(g)\rho_S \quad \forall g \in G$$

$$|+\rangle_C := \frac{1}{\sqrt{|G|}} \sum_{g \in G} |g\rangle_C$$

$$\max_{\sigma \in \text{B-Sym}_G} F(\rho, \sigma)$$



How do we test them?

G-Symmetry

- State ρ_S is G-symmetric $\implies \exists$ purification which is G'-Bose Symmetric

$$\rho_S = U_S(g)\rho_S U_S^\dagger(g) \;\; \forall g \in G$$

$$|\phi\rangle_{S\hat{S}} = \left(\frac{1}{|G|} \sum_{g\in G} U_S(g) \otimes \overline{U}_{\hat{S}}(g) \right) |\phi\rangle_{S\hat{S}}$$

- Purifications are related by unitaries.

How do we pick the right unitary?

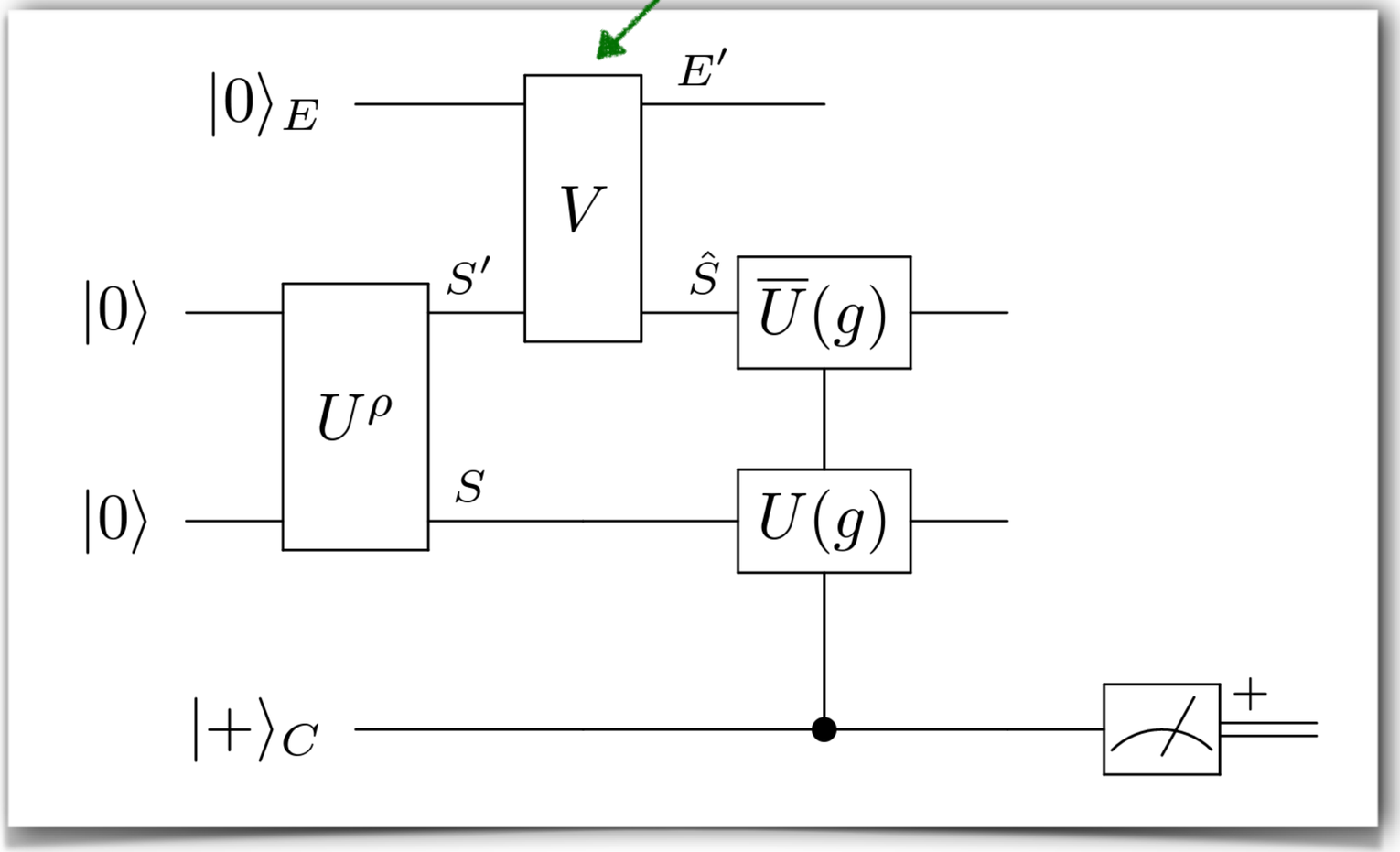
How do we test them?

G-Symmetry

$$\rho_S = U_S(g)\rho_S U_S^\dagger(g) \quad \forall g \in G$$

$$\max_{\sigma \in \mathrm{Sym}_G} F(\rho, \sigma)$$

Replace with parameterized circuit!



Examples

Z-Rotational symmetry

$$G_z = \{R_z(\phi)^{\otimes n}\}_{\phi \in [0, 4\pi)}$$

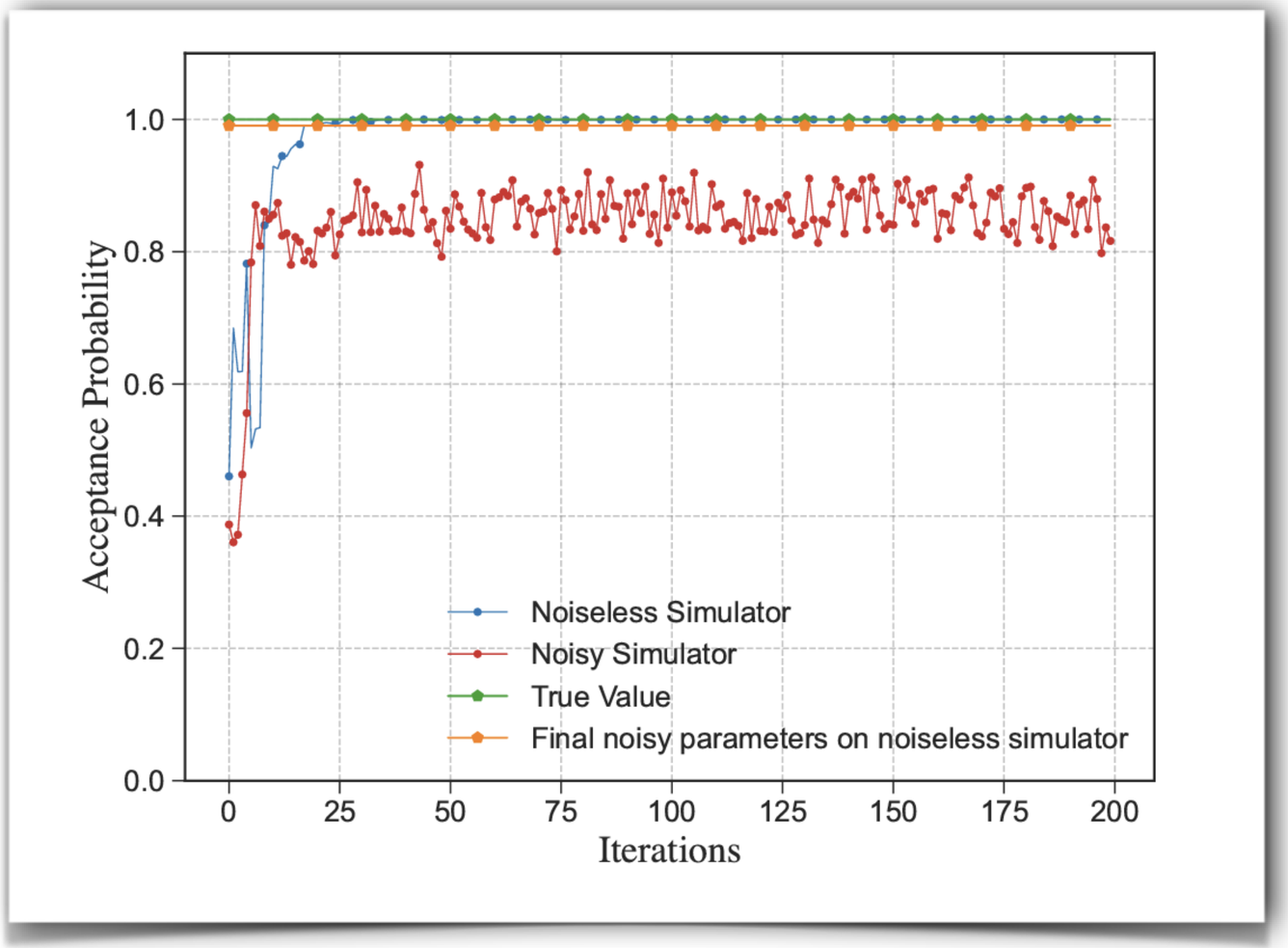
- Related to common reference frames.
- G-Bose symmetry test $\rightarrow$ Test if state has Hamming weight $n/2$.
- G-symmetry test $\rightarrow$ Test if state has fixed Hamming weight.

Examples

Z-Rotational symmetry

- G-symmetry test for

$$|\psi^+\rangle = \frac{1}{\sqrt{2}}\left(|01\rangle + |10\rangle\right)$$



Thank you!